II-3. Circulator Synthesis

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Theoretical consideration of the junction circulator has taken the form of group-theoretical treatment of the characteristic modes of symmetrical junctions[1] and analysis of the spatial configurations of the modes[2] under certain simplifying assumptions regarding the structure of the junction. Although a considerable advance in the quality of Y-junction circulators has taken place during the same period, it has not been possible to apply the results of the theory to the design effort. The mechanical improvements, which include the shaping of the ferrite, use of composite dielectric-ferrite elements, addition of tuning elements at the ports and, in stripline versions, shaping of the center conductor, have increased the complexity of the device to the point where existing simplified theoretical models can serve at most only as a qualitative guide.

This paper considers a network model in which two aspects of circulator performance are singled out as fundamental: nonreciprocal differential phase shift, regarded as taking place in a distributed manner in the region between the ports; and scattering at the ports, regarded as localized and reciprocal. The relevance of this idealized model to existing circulator design is based on the observations: first, that broadband circulation seems to occur only when the electrical distance between ports is at least an appreciable fraction of a wavelength; second, that coupling to the ports, visualized as localized scattering, must involve internal reflections as a fundamental part of circulator action.

This view is embodied in a ring-network representation of the three-port Y circulator illustrated in Fig. 1, in which the elements L are nonreciprocal phase shifters, and the elements T are symmetrical, reciprocal T junctions. The idea of synthesizing a circulator by joining three ferrite phase shifters has been advanced by others,[3,4] but has not received much attention because it has not appeared to offer any practical advantage over existing junction designs. In the present paper, the network is examined more rigorously. Specifically, the role of reflections at the T junctions in determining the characteristics of the circulator is incorporated in a systematic way.

The most significant result of the theory is the discovery that the ring network may exhibit perfect circulation even when the amount of phase differential for propagation around the ring in the two clock senses is very small: a typical value for a favorable case is about 10 degrees per sector. The word "favorable" here has the following meaning: Although circulation can occur for even smaller values of differential phase (a manifestation of the circulator theorem to the effect that *any* nonreciprocal three-port junction can be made to circulate), the action in such cases depends on the existence of rather high standing-wave amplitudes within the ring. From the point of view of low-loss, broadband performance, a network is considered favorable only if it combines a small value of differential phase with small internal wave amplitudes, of the order of unity (times the incident signal).

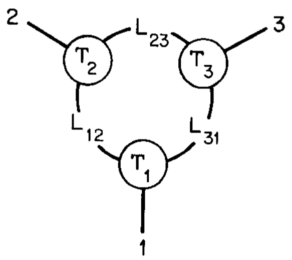

Fig. 1 The ring-network. T and L denote symmetrical T junctions and nonreciprocal phase shifters, respectively.

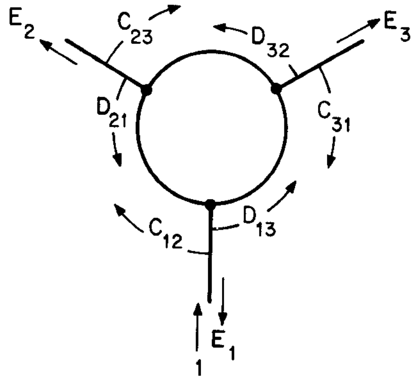

Fig. 2 Internal and scattered waves in the ring network

The theory also yields a prescription for the frequency dependence of scattering at the T's, such that circulation may be made to persist over a band of any width (with, of course, increasing complexity of structure as the bandwidth specification is increased). Thus, if the frequency-dependence of the intrinsic reciprocal and nonreciprocal phases of the ferrite-loaded junction is known, the characteristics of the corresponding T junctions required for circulation over the entire band are completely determined. For the types of differential phase shifter characteristics commonly encountered, it is reasonable to expect that the required T's can be synthesized by the application of known filter principles, resulting in a rigorously flat broadband Y-junction circulator which exhibits only incidental loss and which may be built with extremely efficient use of ferrite. Depending on the application contemplated, this efficiency may be manifested in some combination of compact size, low loss, high peak and average power capacity, or high-speed reversal of the direction of circulation, etc.

The method of carrying out the network analysis is as follows. The scattering coefficients of a symmetrical T junction are expressed in terms of the characteristic modes of the junction. The derivation follows the standard method[5] of the theory of group representations, but the result is obtained with full generality, not found in standard treatments of the subject, so as to represent all physically realizable T junctions satisfying reciprocity, energy conservation and T symmetry. The scattering of a matched, lossless, nonreciprocal phase shifter (denoted by L in Fig. 1) is represented by a reciprocal, or average, phase factor, $\mathcal{E} = \exp[i(\varphi_+ + \varphi_-)/2]$ and a nonreciprocal, or differential, phase factor, $\delta = \exp[i(\varphi_+ - \varphi_-)/2]$, where φ_+ and φ_- refer to clockwise and counterclockwise propagation, respectively. The over-all scattering by the network is then calculated in terms of the parameters of the T's and L's. We obtain the scattering coefficients, E_1, E_2, E_3, denoting reflection, transmission and leakage, respectively, in response to a unit signal incident on port 1 of the ring.

The conditions for perfect circulation are now imposed on the over-all scattering coefficients, E_1, E_2, E_3. They are: input match, $E_1 = 0$; perfect isolation, $E_3 = 0$; lossless transmission, $|E_2| = 1$. These conditions are, of course, not mutually independent; on the contrary, the last condition implies the other two. It turns out to be convenient, however, to use the isolation condition, $E_3 = 0$; solving this, we obtain a biquartic algebraic equation for the reciprocal phase factor, $\mathcal{E}$, and an expression for the nonreciprocal phase factor, δ, in terms of $\mathcal{E}$, both relations involving the scattering coefficients of the T's as parameters. The problem now becomes one of computation.

A program was written for the Lincoln Laboratory IBM 7094-C computer, in which values were assigned to the scattering coefficients of the T's in sets (cases) covering the range of physically realizable T's in a convenient number of steps. For each case, the problem was solved to find: (a) the phase factors $\mathcal{E}$ and δ if they exist; (b) the amplitudes and phases of the internal waves excited by a unit wave incident on port 1 of the ring; (c) the phase of the transmitted wave E_2. In all, about 700 cases were considered. Of these, roughly half gave solutions satisfying all requirements. As an example, the case identified as (36°; 7, 3, 1) is summarized in Table 1. The scattering coefficients r, s, r_d and s_d of the T's are defined in Fig. 3, and the internal partial waves C and D are defined in Fig. 2.

TABLE 1

The Ring-Network Circulator, Case (36°; 7, 3, 1)

	Magnitude	Phase, degrees		Magnitude	Phase, degrees
r	0.244	−45.00	E_2	1.00	14.49
s	0.845	78.21	E_3	0	...
r_d	0.740	27.83	C_{12}	1.23	164.10
s_d	0.476	135.00	C_{23}	1.19	−84.07
$\mathcal{E}$	1.00	41.78	C_{31}	1.19	52.25
δ	1.00	4.55	D_{21}	0.71	−11.41
φ_+	...	46.33	D_{32}	1.19	−127.78
φ_-	...	37.23	D_{13}	1.19	104.99
E_1	0	...			

Note in Table 1 that the scattering coefficients of the T's describe a "poorly" matched junction; nevertheless the over-all match of the ring network is, of course, perfect in this model. The most striking results are the differential phase, $2 \arg \delta = \varphi_+ - \varphi_-$, which is only 9.10 degrees, and the wave amplitudes, which are all of the order of unity.

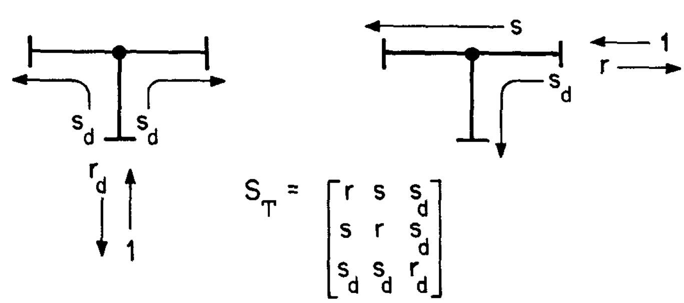

Fig. 3 Definition of the scattering matrix of a symmetrical T junction

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